



Abbots Farm Junior School

CPA Calculation policy for Mathematics Curriculum

This policy supports the White Rose maths scheme used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

- Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a ‘hands on’ component using real objects and is a foundation for conceptual understanding.
- Pictorial representation – a pupil has sufficiently understood the ‘hands on’ experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example $12 \times 2 = 24$.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations.

Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

How to use the policy:

This mathematics policy is a guide for all staff at Abbots Farm Junior School and has been adapted from work by the NCETM and Whiterose. All teachers have been given the scheme of work from the White Rose Maths and are required to base their planning around their year group’s modules and not to move onto a higher year group’s scheme work. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used.

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group’s scheme of work.

Addition

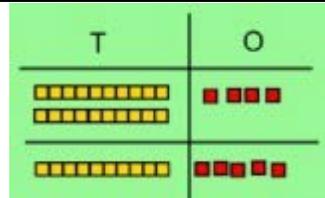
Addition- Year 3

Objective and Strategy

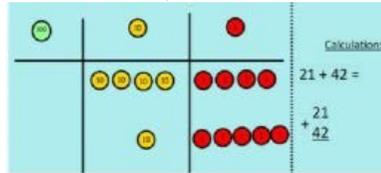
Add and subtract numbers with up to 3-digits, using formal written methods of columnar addition

Column addition (no regrouping)

Concrete



Using manipulatives (dienes, numicon, counters), children are to line up hundreds, tens and ones.

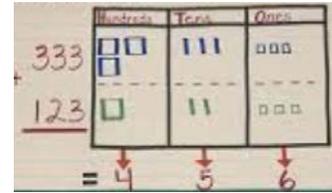


Children should be secure with using PV counters before moving onto pictorial.

The calculation will be shown alongside the model used to see the connection

Model	Calculation

Pictorial



Children are to draw, in a PV frame, the manipulatives, that they are using.

Secure knowledge of representation with the PV columns.

The calculation will be shown alongside the model to see the connection

Model	Calculation

Abstract

$$\begin{array}{r} 223 \\ + 114 \\ \hline \end{array}$$

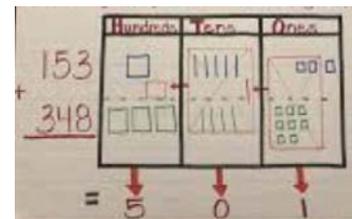
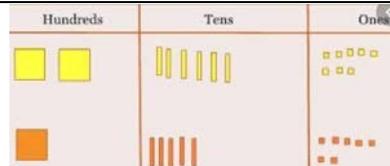
$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

$$337$$

Children to move onto recording more formally.

Some children may need to use the expanded method (see below).

Column addition (with regrouping)



Children can draw a representation of the grid to further support their understanding, carrying the ten *underneath* the line.

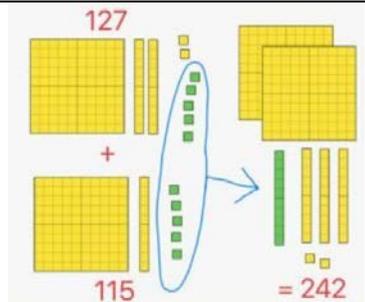
$$20 + 5$$

$$\begin{array}{r} 40 + 8 \\ \hline \end{array}$$

$$60 + 13 = 73$$

Children are to begin with the abstract: expanded form.

For those children, that are confident after AFL, the below method should be used.



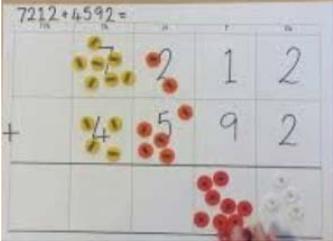
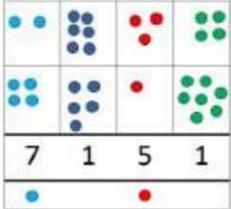
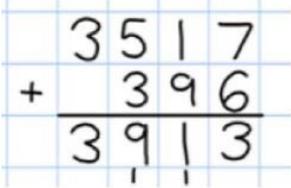
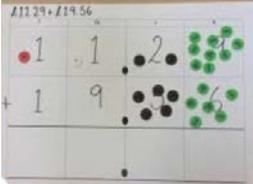
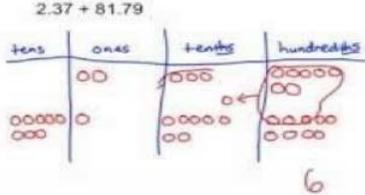
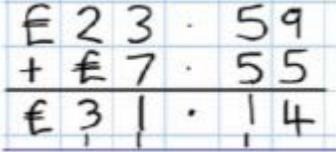
Exchange ten ones for a ten. Model using Dienes, Numicon and place value counters.

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

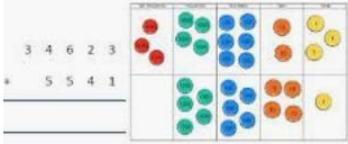
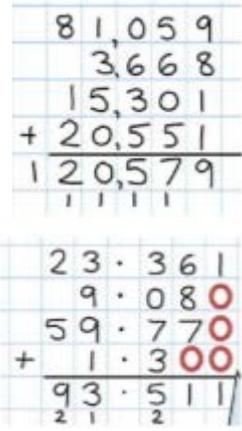
Vocabulary

addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Addition- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract				
<p><i>Using formal written methods of columnar addition where appropriate</i></p> <p>add numbers with up to 4 digits (with exchange)</p>	<p>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p>  <p>The calculation will be shown alongside the manipulative used to see the connection</p> <table border="1" data-bbox="642 662 913 783"> <thead> <tr> <th>Model</th> <th>Calculation</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> </tr> </tbody> </table>	Model	Calculation			 <p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p>
Model	Calculation						
<p>Add decimals with 2 decimal places, including money.</p>	 <p>Introduce decimal place value counters and model exchange for addition.</p>		 <p>As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</p>				
<p>Vocabulary</p>	<p>addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point</p>						

Addition- Year 5/6

Objective and Strategy	Concrete	Pictorial	Abstract
add numbers with more than 4 digits.	See Year 4	See Year 4	 <p>Children should have abstract supported by a pictorial or concrete if needed.</p>
add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	See Year 4	See Year 4	 <p>Insert zeros for place holders.</p>
Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point		

Subtraction

Column
Subtraction
(with
exchanging)

Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.

Column method (using base 10 and having to exchange)

45-26



- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

100	10	1	Calculations
●	●	●	
●●	●●●●	●●●●●	234
			- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

100	10	1	Calculations
●	●	●	
●●	●●●●	●●●●●●●●	234
			- 88

Now I can subtract my ones.

100	10	1	Calculations
●	●	●	
●●	●●●●	●●●●●●	234
			- 88

45
-29

16

Tens | Ones

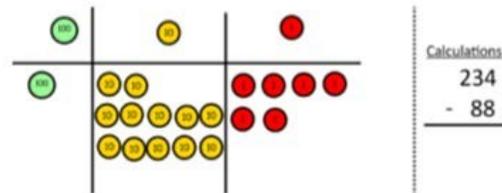
10 + 6 = 16

When confident, children can find their own way to record the exchange/regrouping

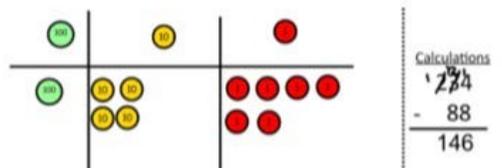
Children should begin with the expanded form. Moving onto a more formal way as below (bottom picture).

$836 - 254 = 582$ <table style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 5px;">800</td> <td style="text-align: right; padding-right: 5px;">130</td> <td style="text-align: right; padding-right: 5px;">6</td> <td></td> </tr> <tr> <td style="text-align: right;">-</td> <td style="text-align: right;">200</td> <td style="text-align: right;">50</td> <td style="text-align: right;">4</td> </tr> <tr> <td style="border-top: 1px solid black; text-align: right;">500</td> <td style="border-top: 1px solid black; text-align: right;">80</td> <td style="border-top: 1px solid black; text-align: right;">2</td> <td></td> </tr> </table>	800	130	6		-	200	50	4	500	80	2		$728 - 582 = 146$ <table style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 5px;">700</td> <td style="text-align: right; padding-right: 5px;">20</td> <td style="text-align: right; padding-right: 5px;">8</td> <td></td> </tr> <tr> <td style="text-align: right;">-</td> <td style="text-align: right;">500</td> <td style="text-align: right;">80</td> <td style="text-align: right;">2</td> </tr> <tr> <td style="border-top: 1px solid black; text-align: right;">200</td> <td style="border-top: 1px solid black; text-align: right;">40</td> <td style="border-top: 1px solid black; text-align: right;">6</td> <td></td> </tr> </table>	700	20	8		-	500	80	2	200	40	6	
800	130	6																							
-	200	50	4																						
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700	20	8																							
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200	40	6																							

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

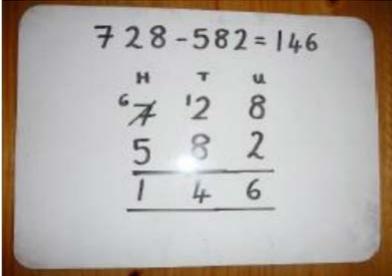


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Vocabulary

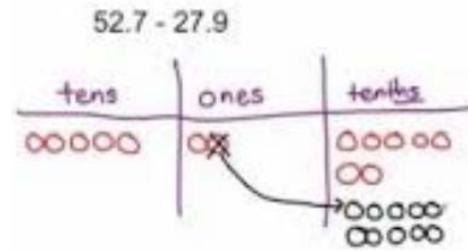
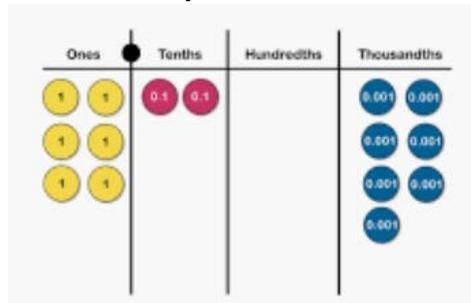
equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Subtraction- Year 4

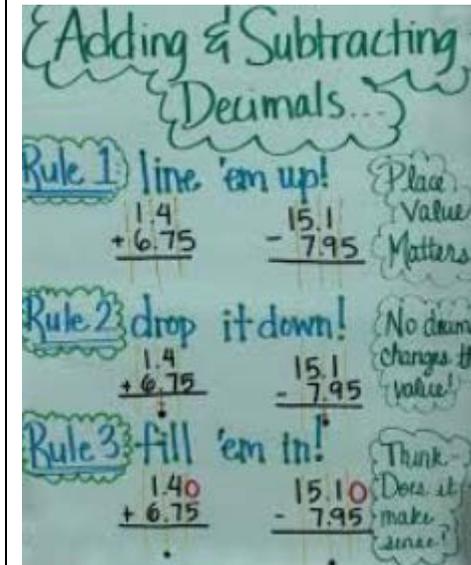
Objective and Strategy	Concrete	Pictorial	Abstract								
<p><i>Subtract numbers with UP to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate</i></p> <p>Year 4 subtraction with up to 4 digits.</p>	<p>Model process of exchange using Numicon, base ten and then move to PV counters.</p> <p>Use the phrase 'take and make' for exchange- see Y3</p> <p>The calculation will be shown alongside the model chosen to see the connection</p> <table border="1" data-bbox="566 1310 837 1428"> <thead> <tr> <th>Model</th> <th>Calculation</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> </tr> </tbody> </table>	Model	Calculation			<p>Children to draw pv counters and show their exchange—see Y3</p> <p>The calculation will be shown alongside the model chosen to see the connection</p> <table border="1" data-bbox="1068 1040 1339 1158"> <thead> <tr> <th>Model</th> <th>Calculation</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> </tr> </tbody> </table>	Model	Calculation			 <p>This will lead to an understanding of subtracting any number including decimals.</p>
Model	Calculation										
Model	Calculation										

Introduce decimal subtraction through context of money

Children to be encouraged to use counters to represent numbers and take counters away to subtract.



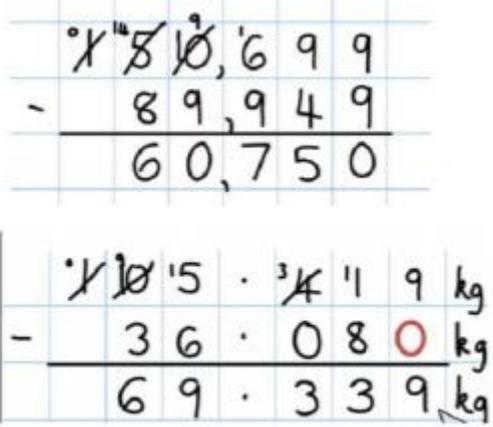
When confident, children can find their own way to record the exchange/regrouping



Vocabulary

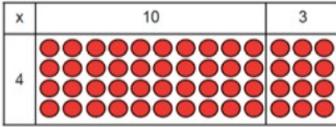
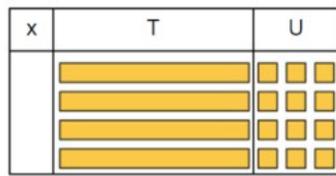
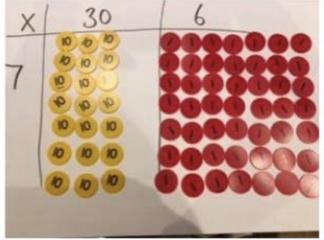
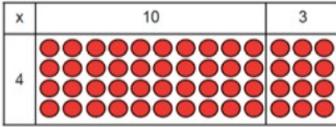
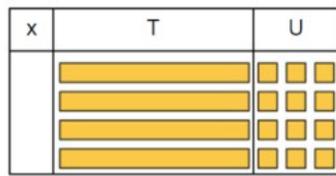
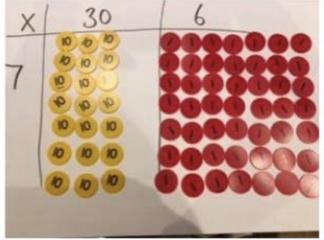
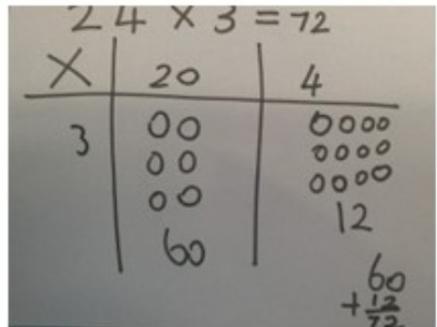
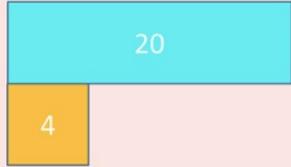
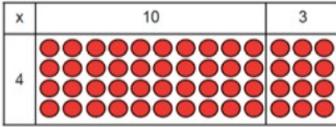
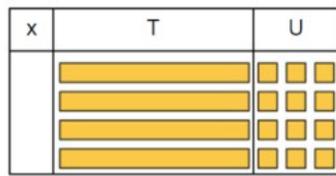
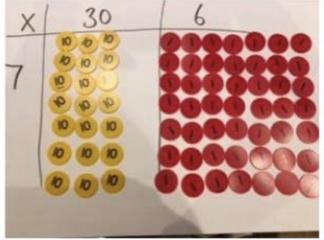
equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Subtraction- Year 5/6

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Subtract with at least 4 digits, including money and measures.</p> <p>Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>See Year 4</p>	<p>See Year 4</p>	
<p>Vocabulary</p>	<p>equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units</p>		

Multiplication

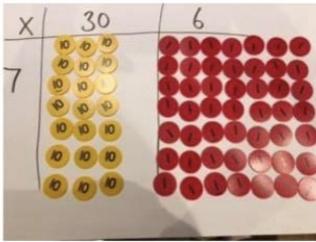
Multiplication- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract																																				
<p><i>Multiplying two digit number by a one digit number</i></p> <p>Grid method progressing to the formal method.</p> <p>Solving problems including missing number problems, integer scaling problems.</p>	<p>Show the link with arrays to first introduce the grid method.</p> <table border="1" style="margin-bottom: 10px;"> <tr> <td style="width: 10px; text-align: center;">x</td> <td style="width: 100px; text-align: center;">10</td> <td style="width: 30px; text-align: center;">3</td> <td rowspan="2" style="font-size: small;">4 rows of 10 4 rows of 3</td> </tr> <tr> <td style="width: 10px; text-align: center;">4</td> <td colspan="2" style="text-align: center;">  </td> </tr> </table> <p>Move on to using Base 10 to move towards a more compact method.</p> <table border="1" style="margin-bottom: 10px;"> <tr> <td style="width: 10px; text-align: center;">x</td> <td style="width: 100px; text-align: center;">T</td> <td style="width: 30px; text-align: center;">U</td> <td rowspan="2" style="font-size: small;">4 rows of 13</td> </tr> <tr> <td style="width: 10px; text-align: center;">4</td> <td colspan="2" style="text-align: center;">  </td> </tr> </table> <p>Move on to place value counters to show how we are finding groups of a number.</p> <table border="1" style="margin-bottom: 10px;"> <tr> <td style="width: 10px; text-align: center;">x</td> <td style="width: 100px; text-align: center;">30</td> <td style="width: 30px; text-align: center;">6</td> </tr> <tr> <td style="width: 10px; text-align: center;">7</td> <td colspan="2" style="text-align: center;">  </td> </tr> </table> <p>Add up each column, starting with the ones making any exchanges needed.</p> <p>The calculation will be shown alongside the model chosen to see the connection</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="width: 50px; text-align: center;">Model</td> <td style="width: 50px; text-align: center;">Calculation</td> </tr> <tr> <td style="height: 30px;"></td> <td style="height: 30px;"></td> </tr> </table>	x	10	3	4 rows of 10 4 rows of 3	4			x	T	U	4 rows of 13	4			x	30	6	7			Model	Calculation			<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> <div style="background-color: #FFDAB9; padding: 10px; margin-top: 10px;"> <p>4 x = 20</p>  </div>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <p>TO x O</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="4" style="text-align: center;">18 x 3 = 54</td> </tr> <tr> <td style="width: 10px; text-align: center;">x</td> <td style="width: 10px; text-align: center;">1</td> <td style="width: 10px; text-align: center;">0</td> <td style="width: 10px; text-align: center;">8</td> </tr> <tr> <td style="width: 10px; text-align: center;">3</td> <td style="width: 10px; text-align: center;">3</td> <td style="width: 10px; text-align: center;">0</td> <td style="width: 10px; text-align: center;">24</td> </tr> </table> <p>Children to add up each column to find the answer.</p>	18 x 3 = 54				x	1	0	8	3	3	0	24
x	10	3	4 rows of 10 4 rows of 3																																				
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x	T	U	4 rows of 13																																				
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18 x 3 = 54																																							
x	1	0	8																																				
3	3	0	24																																				
<p>Vocabulary</p>	<p>Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up</p>																																						

Multiplication- Year 4

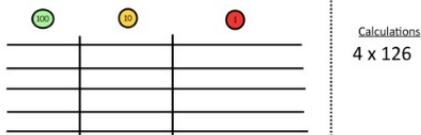
Objective and Strategy	Concret e	Pictori al	Abstra ct
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Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

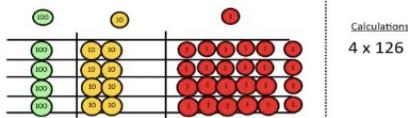


Grid method recap from year 3 for 2 digits x 1 digit

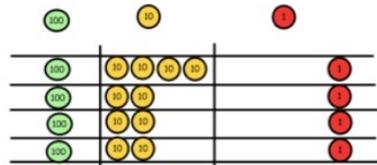
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



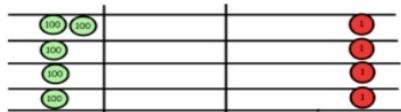
Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



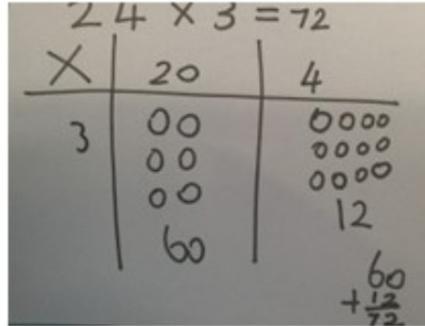
Then you have your answer.



Multiplying numbers by 1 digit (year 4 expectation)

Children can represent their work with place value counters in a way that they understand.

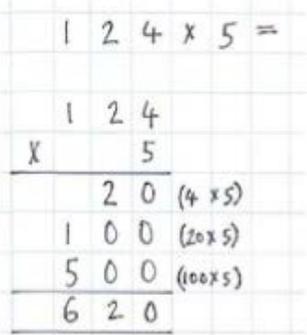
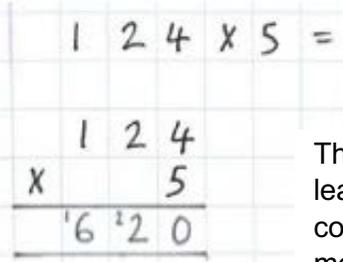
They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



HTO x O

135 x 5 = 675						
X	1	0	0	3	0	5
5	5	0	0	1	5	0
						2
						5

Children to add up each column to find the answer.

<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="309 448 640 842"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p> <table border="1" data-bbox="427 922 698 1040"> <thead> <tr> <th>Model</th> <th>Calculation</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones													Model	Calculation			<p>This grid method may be used to show how this relates to a formal written method.</p> <table border="1" data-bbox="846 363 1256 453"> <tr> <td>x</td> <td>100</td> <td>20</td> <td>4</td> </tr> <tr> <td>5</td> <td>500</td> <td>100</td> <td>20</td> </tr> </table>  <div data-bbox="853 517 1323 874" style="border: 1px solid gray; padding: 5px;"> <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p> <p>8×59</p> <p>$8 \times 60 - 8$</p> <p>$8 \times 6 = 48$</p> <p>$8 \times 60 = 480$</p> <p>$480 - 8 = 472$</p> </div>	x	100	20	4	5	500	100	20	   <p>This may lead to a compact method.</p>
Hundreds	Tens	Ones																												
Model	Calculation																													
x	100	20	4																											
5	500	100	20																											
<p>Vocabulary</p>	<p>Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive</p>																													

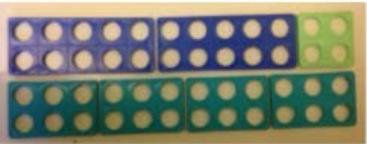
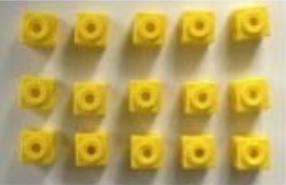
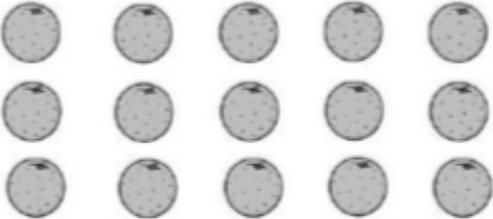
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed
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Multiplication- Year 6

Objective and Strategy	Concrete	Pictorial	Abstract
Multiply decimal up to 2 decimal place by a single digit.			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> $ \begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array} $
vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed		

Division

Division- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  <p style="text-align: center;">20</p> $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Divide 2-digit numbers by a 1-digit number by partitioning into tens and ones using a place value grid

Eva uses a place value grid and part-whole model to solve $66 \div 3$

Tens	Ones
10 10	1 1
10 10	1 1
10 10	1 1

See part-whole model

Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.

Ron uses place value counters to divide 42 into three equal groups

He shares the tens first and exchanges the remaining ten for ones.

Then he shares the ones. $42 \div 3 = 14$

Annie uses a similar method to divide 42 by 3

Tens	Ones
10	1 1 1 1 1
10	1 1 1 1 1
10	1 1 1 1 1

Children may use pictorial representation for the place value counters, alongside the part-whole model

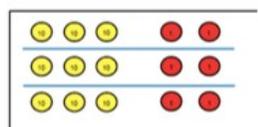
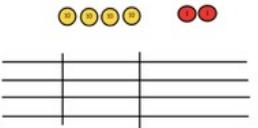
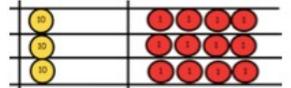
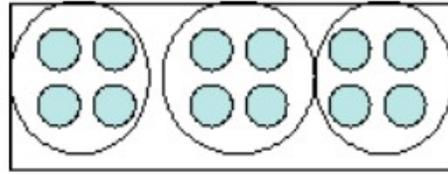
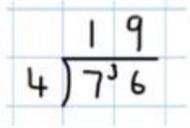
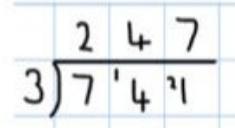
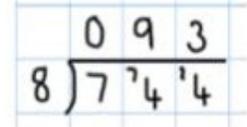
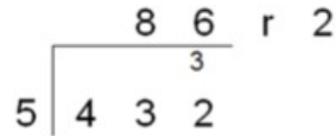
Children use their times-tables to partition the number into multiples of the divisor.

$96 \div 8$
 $96 \div 4$
 $96 \div 3$
 $96 \div 6$

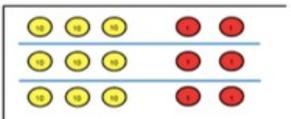
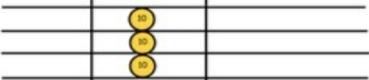
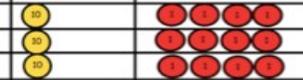
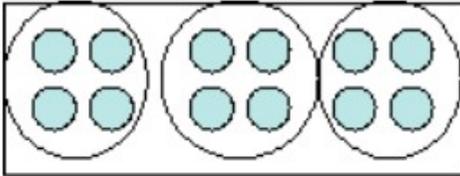
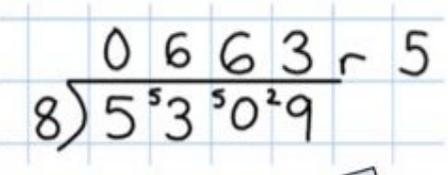
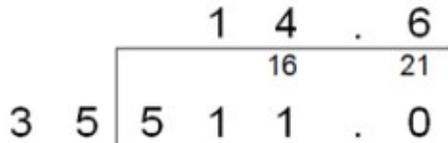
Compare the statements using $<$, $>$ or $=$

$48 \div 4$ $36 \div 3$
 $52 \div 4$ $42 \div 3$
 $60 \div 3$ $60 \div 4$

Division- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract				
<p>Divide up to 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Units</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> </table>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p style="text-align: right; font-size: small;">Calculations $42 \div 3$</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	Tens	Units	3	2	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>Children should be aware that a 0 is used to keep place value, if the number is not divisible.</p> <div style="text-align: center;">  </div> <p>Move onto divisions with a remainder.</p> <div style="text-align: center;">  </div>
Tens	Units						
3	2						
<p>Vocabulary</p>	<p>share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive</p>						

Division- Year 5

Objective and Strategy	Concrete	Pictorial	Abstract
<p><i>Divide at least 4 digit numbers by 1 digit. Interpret remainders appropriately for the context</i></p> <p>Short Division</p>	<p>$96 \div 3$</p> <p style="text-align: center;">Tens Units</p> <p style="text-align: center;">3 2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p style="text-align: right;">Calculations $42 \div 3$</p>  <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>   <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	 <p>Finally move into decimal places to divide the total accurately.</p> 
<p>Vocabulary</p>	<p>share, share equally, one each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.</p>		

Division-Year 6

**Objective
and
Strategy**

**Abstra
ct**

Long Division

Step 1 – a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ \hline 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ \hline 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Step 2 – a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \end{array}$ <p>Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 1 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \downarrow \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Step 3 – a remainder in any of the place values

<p>1. Divide.</p> $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	<p>2. Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	<p>3. Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ -2 \downarrow \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
<p>Divide.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	<p>Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	<p>Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
<p>1. Divide.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	<p>2. Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	<p>3. Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Vocabulary

share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written metho

